WESTERN SYDNEY
UNIVERSITY

Module 3

Element-wise Operations

Element-wise operations

Operation	MATLAB Syntax
Addition	+
Subtraction	-
Multiplication	.*
Division	./
Exponentiation	.^

Element-wise operations

- They are operations that act on corresponding elements between two arrays
- Another way of saying *element-wise* operations is *element-by-element* operations
- Addition and subtraction of arrays is always element-wise
- Multiplication, division, exponentiation of arrays can be elementwise
- Both arrays must be same dimension

Element-wise Operations

- Arrays of numbers in MATLAB can be interpreted as vectors and matrices if vector or matrix algebra is to be applied. Recall that matrices are mathematical objects that can be multiplied by the rules of matrices. To do matrix multiplication, you need to use the standard *, /, and ^ operators [without the preceding . (dot)]. They are *not* for element-wise multiplication, division and exponentiation.

- To deal with arrays on an element-by-element level we need to use the following element-wise or dot-operators:

`.*` , `./` and `.^`

Element-wise Addition and Subtraction

For $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$ and

$B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \end{bmatrix}$

$$A + B = \begin{bmatrix} A_{11} + B_{11} & A_{12} + B_{12} & A_{13} + B_{13} \\ A_{21} + B_{21} & A_{22} + B_{22} & A_{23} + B_{23} \end{bmatrix}$$

$$A - B = \begin{bmatrix} A_{11} - B_{11} & A_{12} - B_{12} & A_{13} - B_{13} \\ A_{21} - B_{21} & A_{22} - B_{22} & A_{23} - B_{23} \end{bmatrix}$$

Elementwise operations

Element-wise Division

- Matrices must be of the same size

For $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$ and $B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \end{bmatrix}$

$$A./B = \begin{bmatrix} A_{11}/B_{11} & A_{12}/B_{12} & A_{13}/B_{13} \\ A_{21}/B_{21} & A_{22}/B_{22} & A_{23}/B_{23} \end{bmatrix}$$

Element-wise Multiplication

- **Not** the same as matrix multiplication in linear algebra
- Matrices must be of the same size

For $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$ and $B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \end{bmatrix}$

$$A .* B = \begin{bmatrix} A_{11} \times B_{11} & A_{12} \times B_{12} & A_{13} \times B_{13} \\ A_{21} \times B_{21} & A_{22} \times B_{22} & A_{23} \times B_{23} \end{bmatrix}$$

Element-wise operations between vectors

If two vectors a and b are: $a = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix}$, then element-by-element multiplication, division, and exponentiation of the two vectors gives:

$$a .* b = \begin{bmatrix} a_1 b_1 & a_2 b_2 & a_3 b_3 & a_4 b_4 \end{bmatrix}$$

$$a ./ b = \begin{bmatrix} a_1 / b_1 & a_2 / b_2 & a_3 / b_3 & a_4 / b_4 \end{bmatrix}$$

$$a .\hat{} b = \begin{bmatrix} (a_1)^{b_1} & (a_2)^{b_2} & (a_3)^{b_3} & (a_4)^{b_4} \end{bmatrix}$$

Element-wise operations between matrices

If two matrices A and B are:

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$$

then element-by-element multiplication and division of the two matrices gives:

$$A .* B = \begin{bmatrix} A_{11}B_{11} & A_{12}B_{12} & A_{13}B_{13} \\ A_{21}B_{21} & A_{22}B_{22} & A_{23}B_{23} \\ A_{31}B_{31} & A_{32}B_{32} & A_{33}B_{33} \end{bmatrix} \qquad A ./ B = \begin{bmatrix} A_{11}/B_{11} & A_{12}/B_{12} & A_{13}/B_{13} \\ A_{21}/B_{21} & A_{22}/B_{22} & A_{23}/B_{23} \\ A_{31}/B_{31} & A_{32}/B_{32} & A_{33}/B_{33} \end{bmatrix}$$

Element-by-element exponentiation of matrix A gives:

$$\mathrm{A} .\hat{} \, n = \begin{bmatrix} (A_{11})^n & (A_{12})^n & (A_{13})^n \\ (A_{21})^n & (A_{22})^n & (A_{23})^n \\ (A_{31})^n & (A_{32})^n & (A_{33})^n \end{bmatrix}$$

Array operations & dot-operators

.*, ./ and .^

- Because scalars are equivalent to a 1×1 array, you can either use the standard or the dot-operators when doing multiplication, division and exponentiation of scalars (i.e., of single numbers).
- It is okay for you to always use the dot-operators, unless you intend to perform vector or matrix multiplication or division.

Example – element-wise multiplication

- Use .* to get element-wise multiplication (notice period before asterisk)
- Both matrices must have the same dimensions

```
>> A = [1 2; 3 4];
>> B = [0 1/2; 1 -1/2];
>> C = A .* B
>> C =
     0   1
     3  -2
```

If matrices not same dimension in element-wise multiplication, MATLAB gives error

```
>> A = [ 1 2; 3 4];
>> B = [1 0]';
>> A .* B % Meant matrix
multiplication!
??? Error using ==> times
Matrix dimensions must agree.
>> A * B % this works
ans =
     1
     3
```

Array vs. Matrix Operations

- Example:

```
x = [2, 1; 3, 4]
y = [5, 6; 7, 8]
```

```
z = x .* y
```

results in [10, 6; 21, 32]; this is array multiplication

```
z = x * y
```

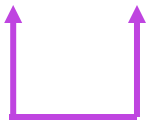
results in [17, 20; 43, 50]; this is matrix multiplication

So, do NOT forget the dot when doing array operations!
(.* ./ .^)

Matrix Multiplication

- Type used in linear algebra
- MATLAB denotes this with asterisk (*)
- Number of columns in left matrix must be same as number of rows in right matrix (inner dimensions)

$$C_{m\times n} = A_{m\times p}\ B_{p\times n}$$



Be careful!

- Be careful when element-wise multiplying square matrices (n×n matrix)
- Both element-wise multiplication and matrix multiplication always work
- If you specify the wrong operator, MATLAB will do the wrong computation and there will be no error!
  - Difficult to find this kind of mistake